CONTROLLING TEMPERATURES IN FREE CONVECTION FOR A HIGH-VISCOSITY FLUID

S. E. Gusev and G. G. Shklover

The self-similarity problem with consideration of the relationship between viscosity and temperature in the range of Prandtl numbers 1-10⁴ has been solved for laminar free-convective flow of a viscous fluid about an isothermal vertical plate. We have calculated the controlling temperatures for local heat transfer, as well as the magnitudes and positions for the maximum of longitudinal velocity, the force of friction, and the thickness of the boundary layer.

A method was first proposed in [1] to reduce the free-convective problem involving variable physical properties to a problem with constant parameters. It was demnostrated there that in the free convection of gases about a vertical plate there exists a unique controlling temperature T_* which corresponds to each physical parameter of the process. Thus, for the average coefficient of heat transfer, its dimensionless magnitude is given by $\theta_* = (T_* - T_0)/(T_w - T_0) = 0.62$, while for the thickness of the thermal boundary layer by level it is 0.02-0.33, for the coefficient of friction at the wall it is 0.9, and for the position of the longitudinal-velocity maximum it is 0.76. A number of studies have been devoted to an investigation of the influence exerted by variable viscosity on heat transfer at $Pr > 1$ [2-7]. Mikheev [2] proposed the introduction of the correction factor $(Pr_0/Pr_w)^{0.25}$. The correction factor $(\nu_0/\nu_w)^{0.21}$ and the controlling temperature $\theta_* = 0.75$ were derived in [3] for the exponential relationship between viscosity and temperature. These values were experimentally confirmed [4]. Analogous results for the relationship between ν and temperature (1) were obtained by an approximate method in [6]. The influence of the variable viscosity on the remaining parameters of the process, to the extent that we are aware, has not been analyzed in detail. It is determined not only by its relative change, i.e., by the quantity $N = \nu_0/\nu_w$, but by the nature of the relationship between ν and temperature. Thus, Carey and Mollendorf [5] assumed that this relationship was linear, while they calculated all of the physical parameters on the basis of the mean boundary-layer temperature. Here it developed that in the range $Pr = 1$-1000, $N = 0.11$-9 deviations from the case of constant physical properties are constant, which indicate the unacceptability of using the mean temperature of the boundary layer as the controlling temperature.

In a number of cases, in order to describe the relationship between temperature and the coefficient of kinematic viscosity we can use the following formula:

$$\nu_0/\nu = 1 + (N - 1)\,\theta. \tag{1}$$

The possibility of its application and the resulting accuracy for specific fluids have been analyzed in [7].

Let us examine the flow of a fluid about an isothermal vertical plate in a Cartesian coordinate system; we will direct the x axis along the plate, and the y axis will be directed along the normal to that plate. On introduction of the dimensionless parameters [8]

$$\eta = \frac{Gr_x^{0.25}}{x} \int_0^y \rho/\rho_0 \, dy, \quad \psi = \nu_0 \, Gr_x^{0.25} f(\eta)$$

the problem reduces to one that is self-similar.

We have to solve the following system of equations:

$$(\mu/\mu_0 f'')' + 0{,}75 ff'' - 0{,}5 (f')^2 + \theta = 0,$$
$$\theta'' + 0{,}75\, Pr_0 \theta' f = 0 \tag{2}$$

for the boundary conditions $\eta = 0$, $f = f' = \theta - 1 = 0$, $\eta = \infty$, $f' = \theta = 0$. The solution is found by the numerical Newton method, where in each iteration system (2) is integrated by the Runge—Kutta—Gilles method. As the initial approximation we employed results obtained through a semiintegral method [6], which usually has made it possible to achieve convergence of the values of $f''(0)$ and $\theta'(0)$ with an accuracy to the sixth figure after the decimal point within several iterations.

TABLE 1. Values of f''(0)

N	Pr_0				
	1	10	100	1000	10000
0,1	0,14548	0,086393	0,049649	0,028179	0,015888
0,2	0,26625	0,16091	0,093144	0,052930	0,029862
0,5	0,55229	0,34680	0,20399	0,116543	0,065870
1	0,90819	0,59283	0,35595	0,20497	0,11617
2	1,43994	0,97721	0,60322	0,35217	0,20064
5	2,56276	1,81638	1,16355	0,69919	0,40381
10	3,89490	2,86324	1,86611	1,14987	0,67601

TABLE 2. Values of $-\theta'(0)$

N	Pr_0				
	1	10	100	1000	10000
0,1	0,31005	0,60718	1,1142	2,0013	3,5699
0,2	0,32970	0,64963	1,1946	2,1471	3,8310
0,5	0,36693	0,73715	1,3654	2,4605	4,3936
1	0,40103	0,82684	1,5495	2,8040	5,0143
2	0,43675	0,93189	1,7784	3,2440	5,8180
5	0,48179	1,08511	2,1361	3,9692	7,1757
10	0,51110	1,20911	2,4360	4,6130	8,4320

The integration interval is E = 0.05 for Pr ≤ 100 and E = 0.01 for Pr = 10^3-10^4; η_{max} = 20 for 1 ≤ Pr ≤ 100 and η_{max} = 50 for Pr ≥ 1000. The results are shown in Tables 1 and 2.

Based on the values found for $\theta'(0)$ we calculated the determining controlling temperatures for the local coefficient of heat transfer. They are equally valid for an isothermal plate as for the mean values. It must be kept in mind here that when we calculate the coefficient of kinematic viscosity on the basis of the controlling temperature T_v, in the isothermal case it is not only the Grasshof number Gr_x that changes, but also the Prandtl number $\mathrm{Pr}_v = \nu(T_v)/a$. It is therefore necessary to choose T_v so as to satisfy the condition

$$\alpha_{\mathrm{const}}\,(\mathrm{Pr}_v) = \alpha_{\mathrm{var}}\,(\mathrm{Pr}_0,\ N). \qquad (3)$$

Here α_{const} is the coefficient of heat transfer in the presence of constant physical properties and the Prandtl number Pr_v, α_{var} is the coefficient of heat transfer in the case of variable physical properties, and in this case the Prandtl number is equal to Pr_0 at some distance from the wall.

Relationship (3) leads to the equation

$$(\mathrm{Pr}_v/\mathrm{Pr}_0)^{0,5} = \theta'\,(0)|_{\mathrm{Pr}_v,\ N=1}\,/\theta'\,(0)|_{\mathrm{Pr}_0,\ N}\,. \qquad (4)$$

The familiar function of the Prandtl number for constant physical properties [9] is found in the numerator of the right-hand side:

$$\theta'\,(0)|_{\mathrm{Pr}_v,\ N=1} = H\,(\mathrm{Pr}_v) = \frac{3}{4}\left(\frac{\mathrm{Pr}_v^2}{2,43478 + 4,884\,\mathrm{Pr}_v^{0,5} + 4,95283\,\mathrm{Pr}_v}\right)^{0,25}.$$

Having solved Eq. (4) and having determined the Prandtl number Pr_v, taking relationship (1) into consideration, it becomes possible to find the controlling temperature θ_* for the local (and average) heat transfer:

$$\theta_* = (\mathrm{Pr}_0/\mathrm{Pr}_v - 1)/(N - 1). \qquad (5)$$

The values of θ_* shown in Fig. 1 have been calculated in accordance with (5). Basically, they fall within the range θ_* = 0.75-0.85. The complex nature of the change in θ_* can be ascribed to the influence of two contradictory factors: viscous friction and inertial effect. As was demonstrated in [10], taking into consideration the variability of ν in the absence of inertial terms leads to the monotonic rise of θ_* as Pr_0 increases.

For purposes of comparison the figure shows the controlling temperatures, equivalent to the introduction of the correction factor $(\nu_0/\nu_w)^m$, where m = 0.21, and these are obtained if the right-hand side of (4) is replaced by this factor. For $\mathrm{Pr}_0 > 10^3$ the controlling temperatures calculated by a numerical method and determined on the basis of the correction factor, when N > 1 are close to one another. Numerical calculation shows that over the entire range of change in N and Pr_0 it is impossible to describe the influence of the variable viscosity by means of a correction factor with a constant m. An analogous conclusion is reached by
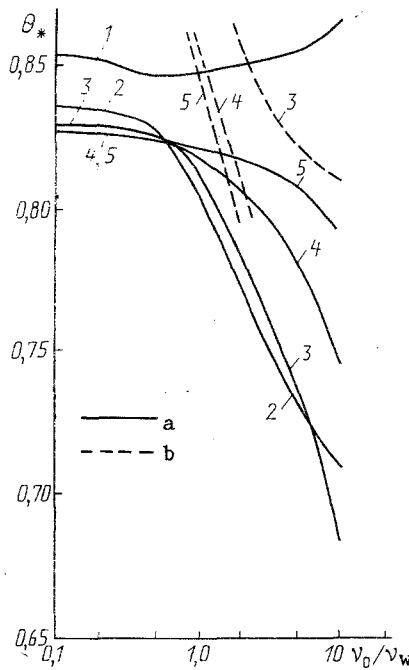
Fig. 1. Temperature determining the Nusselt number; the Prandtl number: 1) 1; 2) 10; 3) 10^2; 4) 10^3; 5) 10^4; numerical calculation of system (2) (a) for the correction factor $(\nu_0/\nu_w)^{0.21}$ (b).
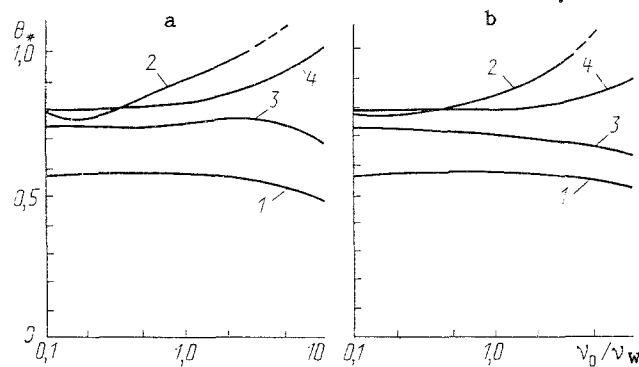

Fig. 2. Definitive temperatures for: 1) maximum longitudinal velocity u_{max}; 2) distance from wall to the point of maximum longitudinal velocity δ_m; 3) thickness of thermal boundary layer based on level $0.01\delta_T$; 4) force of friction F_{fr} at the wall; a) $Pr_0 = 100$; b) $Pr_0 = 1000$.

the authors of [11], where it is recommended when N > 10 to reduce the factor m to 0.17, which is in agreement with the nature of the change in the controlling temperature in our calculations. Usually, in actual practice the controlling temperature is assumed to be constant. Let us find the error associated with such an assumption. For practical purposes, we can recommend the value of $\theta_{theo} = 0.82$, which makes it possible for the entire investigated range to achieve satisfactory accuracy. The maximum error when $Pr_0 = 100$, N = 10 amounts to 3.5%. With $Pr_0 = 1$ the error ranges from —0.4% to +2.6%. The value of $\theta_* = 0.75$ recommended in [4] works better in the region N > 1, where the error does not exceed 1.7%. When N < 1 the accuracy is lower, the error increases to 6.4% for N = 0.1, $Pr_0 = 10$.

A similar calculation of the controlling temperature was undertaken for other parameters of the process:

the maximum magnitude of the longitudinal velocity

$$u_{max} = v_0/x \, Gr_x^{0.5} \, f'_{max} (Pr_0, \, N),$$

the force of friction at the wall

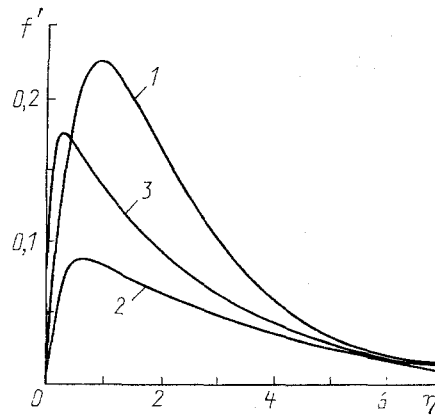$$F_{fr} = \mu_w v_0/x^2 \, Cr_x^{0.75} f'' (0)|_{Pr_0, \, N},$$

Fig. 3. Profile of longitudinal velocity, constant viscosity: 1) Pr = 10; 2) Pr = 100; 3) variable viscosity: $\nu_0/\nu_w = 10$, $Pr_0 = 100$.

the thickness of the thermal boundary level at the 0.01 level

$$\delta_t = y = x\,(Gr_x)^{-0.25}\,\eta_{0,01},$$

the distance from the wall to the point of the maximum in the longitudinal velocity δ_m. The expression for δ_m is analogous to δ_T. For the thickness δ of the dynamic boundary layer, calculated with respect to the $0.01u_{max}$ level, we can use the following approximate relationship: $\delta \simeq C\sqrt{Pr_0}\delta_t$, where $C = 0.7$ for $1 < Pr_0 < 10^3$ and $C = 1$ for $Pr_0 = 10^4$. The corresponding controlling temperatures can be seen in Fig. 2. An unexpected result is found for certain of the parameters in that with an increase in N the controlling temperature becomes greater than unity, which contradicts general opinion with regard to its sense. This can be explained by the nonmonotonic nature of the effect resulting from the variability of viscosity.

Let the quantity ν within the limits of the boundary layer change from ν_w (at the wall) to ν_0 (at some distance from the wall). We will perform three calculations: two with constant physical properties for ν_0 and ν_w and the third, where viscosity varies from ν_0 to ν_w. If the parameter of the process for the variable physical properties falls between the corresponding values for the constant properties, then the controlling temperature is $0 < \theta_* < 1$. For example, Fig. 3 shows three velocity profiles: two for constant properties (N = 1, Pr = 100, $\nu = \nu_0$ and Pr = 10, $\nu = \nu_w$) and one for the variable N = 10, $Pr_0 = 100$, for which $\nu_w \le \nu \le \nu_0$. The maximum velocity falls between the values for the constant properties, and for this the controlling temperature is less than unity. And here the quantity δ_m is smaller, and F_{fr} is larger, than each of the values for the constant properties, as a consequence of which the controlling temperatures for these values are greater than unity.

In a comparison of the experimentally measured values of the velocity (or temperature) profiles under conditions of variability an attempt is usually made to compare their properties with the theoretical values for the constant physical properties. It is implicitly assumed here that in all of the cases the profiles are similar and that they can be combined, through selection of appropriate scales. For example, for the velocity scale we use the value of the coefficient of kinematic viscosity, calculated from the controlling temperature. Such a procedure occasionally leads to divergence between experimental and theoretical values for velocity in the region of its maximum (see, for example, [8, 12]). As we can see from Fig. 3, since the velocity profiles in the case of variable properties are not similar to the profiles in the case of constant physical properties, such divergence is fundamental and cannot be eliminated simultaneously through the entire thickness of the boundary layer.

CONCLUSIONS

We have derived a self-similar solution for the free convection of a viscous fluid around a vertical isothermal plate where viscosity varied over a range of Pr = 1-10⁴.

We have calculated the controlling temperatures for the local and average heat transfer, the magnitude and position of the maximum in longitudinal velocity, the force of friction, and the thickness of the boundary layer.

It has been established that for certain parameters of the process the dimensionless controlling temperature becomes greater than unity, and this is explained by the nonmonotonic relationship between these parameters and the coefficient of kinematic viscosity. It has been demonstrated that for a fluid with variable viscosity it is fundamentally impossible, through introduction of the controlling temperature, to refer the profile of the longitudinal velocity to the viscosity case.

NOTATION

ψ, stream function, m²/sec; ν, coefficient of kinematic viscosity, m²/sec; T, temperature, °C; $N = \nu_0/\nu_w$; μ, coefficient of dynamic viscosity; Pr, Prandtl number; f, dimensionless stream function; η, self-similar variable; $Gr_x = g\beta(T_w - T_0)x^3/\nu_0^2$, Grasshof number; ρ, density of the fluid, kg/m³; α, heat transfer coefficient, W/(m²·K); δ_t, thickness of thermal boundary layer; F_{fr}, force of friction. Subscripts: 0, temperature of the fluid at some distance from the wall; w, temperature of the wall; *, controlling temperature; max, maximum longitudinal velocity.

LITERATURE CITED

1. E. M.Sparrow and J. L. Gregg, *Trans ASME*, **80**, No. 4, 879-886 (1958).
2. I. M. Mikheeva, in: *Heat Transfer and Thermal Modeling* [in Russian], Moscow (1959), pp. 226-238.
3. S. Akagi, *Trans. Jpn. Soc. Mech. Engnr.*, **30**, No. 213, 624-635 (1967).
4. T. Fujii, M. Takeuchi, M. Fujii, et al., *IJHMT*, **30**, No. 5, 753-787 (1970).
5. V. P. Carey and J. Mollendorf, in: *Proc. 6th Int. Transfer Conf.*, Toronto, Vol. 2 (1978), NC-5, pp. 211-216.
6. G. G. Shklover and S. E. Gusev, *Inzh.-Fiz. Zh.*, **53**, No. 2, 229-236 (1987).
7. S. E. Gusev, "Heat exchange and hydrodynamics in the free-convective flow of a coolant with variable physical properties in a horizontal tube bundle," Candidate's Dissertation, Technical Sciences, Kaluga (1985).
8. H. Shaukatullah and B. Gebhart, *Numer. Heat Transfer*, **2**, No. 2, 215-232 (1979).
9. I. Dzhaluriya, *Natural Convection. Heat and Mass Exchange* [Russian translation], Moscow (1983).
10. G. G. Shklover and S. E. Gusev, in: *Convective Flows: Collection of Scientific Papers* [in Russian], Perm' (1985), pp. 92-98.
11. A. Z. Shcherbakov, *Transport and Storage of High-Viscosity Petroleum and Petroleum Products with Heating* [in Russian], Moscow (1981).
12. Yu. A. Sokovishin and A. A. Kovkova, *Inzh.-Fiz. Zh.*, **48**, No. 1, 50-64 (1985).

DETERMINING THE MAXIMUM VELOCITY OF A GAS
BEING DISCHARGED OUT OF A NOZZLE

V. V. Skryabin

UDC 629.7.036.53

It is proposed to calculate the maximum discharge velocity of a chemically neutral real gas out of a Laval nozzle with consideration given to the latent heat of vapor formation of the substance in the expansion of the gas within the nozzle to total condensation, as well as to calculate the parameters which correspond to the saturation line. An equation has been derived to determine the maximum velocity of the discharge gas from an energy point of view.

The velocity at which a gas is discharged out of the diffuser of a Laval nozzle as the gas expands to a pressure of $p_n = 0$ and a temperature $T_n = 0$ and, consequently, given a gas enthalpy of $i_n = 0$ this discharge velocity is usually referred to as the maximum velocity [1]. The value of this maximum velocity is determined from the equation of conservation for the energy of the gas as the latter expands within a nozzle, without phase transition:

$$W_1^2 = 2i_c. \tag{1}$$

With $T_c = $ const and a variable p_c for an ideal gas $i_c = $ const, and, therefore, in actual practice, when $T_c = $ const, the value of W_1 is assumed to be independent of p_c [2].

Let us examine the question as to whether the velocity defined by Eq. (1) is the only maximum velocity for a chemically neutral real gas. A basis for the validity of the formulation of such a problem can be found, as demonstrated in [3, 4], that the heat of vapor formation evolved in the condensation of the metal vapors generates (within the diffuser of the nozzle) an increase in the velocity of the uncondensed gas.